Short-run Price-Inventory Dynamics in Crude Oil Market

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Abstract This paper explores the short-run price-inventory dynamics in the presence of different shocks. Classical competitive storage model states that inventory decision considers both current and future market condition, and thus interacts with spot and expected future spot prices. We study competitive storage holding in an equilibrium framework, focusing on the dynamic response of price and inventory to different shocks. We show that news shock generates response profile different from traditional contemporaneous shocks in price and inventory. The model is applied to world crude oil market, where the market expectation is estimated to experience a sharp change in early 2000s, together with a persisting constrained supply relative to demand. The expectation change has limited effect on crude oil spot price though.

Keywords: crude oil spot price; crude oil supply; crude oil demand; crude oil inventory; news shock

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1 Introduction

Inventory behavior is usually linked to the expectation about future prices. In the discussion of the causes of the recent crude oil price uprising, one key question is whether speculation played an important role in driving up the oil price during 2007-2008. Regardless of their stand on it, researchers turn to inventory for a better understanding of the speculative or precautionary incentive in the oil market, as anticipation of future increases in oil price could lead to speculative inventory increase. Following this intuition, Kilian and Murphy (2010) argue against a major role of speculation where the authors identify the forward-looking element of the real price with data on oil inventories. Hamilton (2009b) proposes a slightly different argument that if speculation drives up the spot price, the inventory would be accumulated as a result of the spot price being higher than its intrinsic value. However, Parsons (2009) points out that expectation of higher future price doesn't necessarily lead to inventory accumulation if the entire term-structure is elevated, implying that a major role of speculation might not be easily ruled out by simply using inventory data.

We propose to study the inventory-price dynamics in an equilibrium model in which inventories, sales and prices are determined endogenously under rational expectation. Inventory could be especially helpful in understanding the short-run price dynamics and sources of shocks driving the price variations in general, as different shocks could potentially generate different inventory-price dynamics. In their extension of the canonical competitive storage model (Wright and Williams (1982), Deaton and Laroque (1992, 1996)), Dvir and Rogoff (2010) show that the price effects of storage arbitrage and the resulting price distribution depend on the types of shocks in the market. Theoretically, in the presence of persistent growth shocks to the supply/demand, rational storage behavior might actually enhance price volatility, which enriches earlier understanding of competitive storage's price-smoothing effect (Wright and Williams (1982)).

Kilian (2009) points out that oil market is not exogenous and isolated from other parts of the economy. Oil prices dynamics is subject to the macroeconomic shocks as much as the economy is to oil price shocks. In response to Deaton and Laroque (1992, 1996)'s critique, Arseneau and Leduc (2012) improves empirical performance of classical competitive storage model by embedding it into a general equilibrium framework. While in this paper we don't attempt to model the price-inventory dynamics in a general equilibrium context, we do consider inventory decision and the resulting price as a process subject to shocks of various origins from both inside and outside the oil market. Discovery of new oil fields serves as a supply shock which could be temporary or persistent depending on the size of the field. Advances in engineering in general would be a technology shock to the overall economy that also affects the supply side of oil market persistently. A severely cold winter boosts the specific demand for oil temporarily. A fast-growing economy with limited alternative energy sources means persistent strong demand for oil. The equilibrium context enables us to study the dynamic price response to different shocks in the presence of inventory.

In this paper we show that the short-run price dynamics implied from rational storage behavior in a competitive market indeed depends on the nature of shocks. While short-lived shocks which lead to current price increase suppresses the incentive to hold inventory in the current period, the inventory-holding incentive is not affected as much under persistent shocks which lead to both current and future spot price increases. News shocks about future market condition which don't affect current price contemporaneously generate yet a different response in inventory-holding. The different price-inventory dynamics would then help uncover the shocks driving behind the observed data.

Our model follows the tradition of Blinder and Fischer (1981), Eichenbaum (1984), Pindyck (1994) and related literature. While earlier research in this strand of literature mostly focuses on the role of inventory as a buffer stock in the business cycle and has a more macroeconomic perspective, like the study of storable output by Blinder and Fischer (1981) and the study of automobile industry by Blanchard (1983), these models also have implications for the price-inventory relation for the purpose of estimation. For example, Eichenbaum

(1984) studies the inventory of finished goods in several industries, using inventory and price data to estimate the cost structure of inventory-adjustment. Pindyck (1994) studies the the cost of inventory-holding and its implications for inventory and price behavior.

Observing that crude oil market doesn't typically experience stock-outs, we follow Eichenbaum (1984) and Pindyck (1994) in that we don't model stock-out explicitly. Instead, we model a non-linear marginal convenience yield as Pindyck (1994) so that when the future spot price is expected much lower than the current, the inventory would be drawn down, yielding extremely high marginal convenience yield. We also add to the work of Pindyck (1994) the endogenously determined price in a competitive market and an inventory-adjustment cost. This equilibrium model allows us to study the dynamics of inventory and price in response to different shocks. Interested especially in uncovering the shock processes, we solve and estimate the model following the macroeconomic literature, compromising the potential insights on the inventory-holding cost structure as considered by Eichenbaum (1984) and Pindyck (1994).

The paper is planned as follows. Section 2 introduces the model. Section 3 discusses the theoretical implications on the inventory-price dynamics in an equilibrium model under rational expectation. Section 4 presents the estimation result and discussion of the shocks in the context of 07-08 oil price spike. Section 5 carries out robustness check of the estimates. Section 6 tests the forecasting ability of the model. Section 7 concludes.

2 The Model

2.1 Inventory Decision

A profit-maximizing oil producer in a competitive market makes decision with regards to its inventory-holding following the condition:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \tag{1}$$

Inventory decision N_{t+1} at time t would be such that the resulting net marginal cost of holding inventory $E_t[MIC_{t+1}]$ would be just covered by the expected intertemporal price difference $\beta E_t[P_{t+1}] - P_t$.

The net marginal cost of holding inventory here includes all costs and benefits associated in general. Inventory facilitates production and delivery scheduling and avoids stockouts in the face of fluctuating demand and changing technology shocks. These benefits motivate producers to hold inventory even if they expect the price to fall, as discussed in Brennan (1958). Pindyck (1994) further proposes a exponential form for the net marginal cost based on the observation that the scatter plot of relative inventory against the net marginal cost of storage is nonlinear (Pindyck (1994)) [1]. We follow this functional form, assuming that the net marginal cost is affected by the current price and inventory relative to the quantity demanded positively. Another visual feature is that the relative inventory data is much less volatile compared to the price after the seasonality in inventory is taken away. Thus we introduce an inventory adjustment cost to Pindyck (1994), following earlier literature like Eichenbaum (1984).

$$MIC_{t+1} = P_t * [\alpha(\frac{N_{t+1}}{N_{t+1} + Y^s_{t+1} - N_{t+2}})^{-\phi} + \delta + \Delta(\frac{N_{t+1}}{N_t}) - \beta * \Delta(\frac{N_{t+2}}{N_{t+1}})] \tag{2}$$

The net marginal cost of storage here takes into consideration the physical cost of holding inventory δ, the intangible benefit of inventory-holding to avoid stock-out (the first part) and the inventory adjustment costs (Δ of relative inventory changes) for both current and next periods. α is constrained to be negative, such that the intangible benefit would be lower when the inventory level is high relative to demand. Δ is assumed to be zero in the steady state when there's no change in inventory.

[1] The net marginal cost of storage, or the negative net marginal convenience yield is inferred using spot and futures prices in Pindyck (1994)

2.2 Demand for Oil

Kilian (2009) argued that the overall economic performance affects price as much as the specific demand for oil. Let Y_t^d denote a measure of overall economic performance, which can be thought of as some function of world GDP, or the index of world economic activities as proposed by Kilian (2009). The inverse demand function of oil depends on both quantity demanded Q_t^d and the overall economic performance Y_t^d as follows:

$$P_t = P(Q_t^d, Y_t^d)$$

which is decreasing in Q_t^d and increasing in Y_t^d. We further posit this inverse demand function to be homogeneous of degree zero, i.e. only the consumption relative to the overall economic performance matters, as oil consumption and world economic performance is highly correlated. We use a CES inverse demand function:

$$P_t = c(\frac{Q_t^d}{Y_t^d})^{-\frac{1}{\gamma}}$$

where c is a scalar. Crude oil consumption Q_t^d is the crude oil production Q_t^s less the change in inventory $N_{t+1} - N_t$, and finally we have:

$$P_t = c(\frac{N_t + Q_t^s - N_{t+1}}{Y_t^d})^{-\frac{1}{\gamma}} \tag{3}$$

2.3 Supply of Oil

On the supply side of the market, we consider the log of world crude oil supply as a random walk process with a drift.

$$\log(Q_t^s) = \log(Q_{t-1}^s) + \log(\mu_t^s) \tag{4}$$

$$\log(\mu_t^s) = \bar{\mu} + \epsilon_t^\mu \sim N(0, \sigma_\mu^2) \tag{5}$$

The demand shock and supply shock are isomorphic in the model. The supply growth rate shock $\log(\mu_t^s)$ affects price through Q_t^s, as modeled in the inverse demand function. In terms of its effect on price, μ_t^s can be effectively thought of as either newly discovered oil sources (a supply shock) or economically-feasible alternative energy sources (a demand shock). For this reason, we model the supply relative to overall economic performance, $\frac{Q_t^s}{Y_t^d}$, instead of modeling demand explicitly.

We complete the model with the following assumptions about stochastic processes driving behind the relative supply, $\frac{Q_t^s}{Y_t^d}$:

$$\log \frac{Q_t^s}{Y_t^d} = y_t^\tau + y_t^c \tag{6}$$

$$y_t^\tau = \rho^\tau y_{t-1}^\tau + n_{t-1}^\tau + \epsilon_t^{y_\tau} \qquad \epsilon_t^{y_\tau} \sim N(0, \sigma_{y_\tau}^2) \tag{7}$$

$$y_t^c = \rho^c y_{t-1}^c + \epsilon_t^{y_c} \qquad \epsilon_t^{y_c} \sim N(0, \sigma_{y_c}^2) \tag{8}$$

$$n_t^\tau = \rho^{n_\tau} n_{t-1}^\tau + \epsilon_t^{n_\tau} \qquad \epsilon_t^{n_\tau} \sim N(0, \sigma_{n_\tau}^2) \tag{9}$$

We considers three types of shocks: persistent shock y_t^τ, temporary shock y_t^c, and news about persistent shock n_t^τ. As Dvir and Rogoff (2010) have shown, the persistence of the shocks matters to the price dynamics. In addition to the persistent and temporary shocks, we're especially interested in the role of news shock in the price-inventory dynamics. When the market believes in the news about the future, even though the relative supply isn't affected in the current period, rational market participants would still respond right away to the news by adjusting inventory and quantity demanded. This news shock serves as our attempt to model the speculative incentive in the market: the market believes the price would be higher in the future and respond to it rationally. As we will show later in the simulation of the model, these three different shocks generate different profiles of price-inventory dynamics over time. Such "patterns" of dynamics would help us identify the shocks driving the recent oil price fluctuations, when we bring the model to data.

2.4 Equilibrium

In this model, the world supply of crude oil is not a stationary process. In order to solve for the steady state, we follow the macroeconomic literature in treating the variables with a trend.

We normalize inventory by world supply, $n_{t+1} = \frac{N_{t+1}}{Q_t^s}$. The normalized inventory variable n_{t+1} can be thought of as the "effective" inventory level. We also denote the relative supply by lower letter, $q_t^s = \frac{Q_t^s}{Y_t^d}$. The model then can be rewritten in terms of effective inventory n and relative supply y:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \tag{10}$$

$$MIC_{t+1} = P_t * [\alpha(\frac{n_{t+1}/\mu_{t+1}^s}{n_{t+1}/\mu_{t+1}^s + 1 - n_{t+2}})^{-\phi} + \delta + \Delta(\frac{n_{t+1}}{n_t/\mu_t^s}) - \beta * \Delta(\frac{n_{t+2}}{n_{t+1}/\mu_{t+1}^s})] \tag{11}$$

$$P_t = c[(n_t/\mu_t^s + 1 - n_{t+1}) * q_t^s]^{-\frac{1}{\gamma}} \tag{12}$$

along with the exogenous processes μ_t^s, y_t^τ, y_t^c and n_t^τ given by equations 5 6 7 8 9. Now $\{y_t\}$ is a stationary process.

Taking as given the exogenous shocks μ_t^s, y_t^τ, y_t^c, n_t^τ and the resulting y_t, and an initial stock of effective inventory n_0, the equilibrium of the model is a sequence of $\{P_t, n_{t+1}\}$ that satisfies: the optimality conditions of the oil producing firm 10 and 11; the market clearing condition 12.

3 Solving the Model

In this section we solve the model and study the implications of the model on short-run price-inventory dynamics as the baseline analysis. To solve the model for a given set of parameters, we log-linearize the model around its deterministic steady state and solve the resulting linear rational expectations model as in Blanchard and Kahn (1980). The resulting linearized model links the price time series and the effective inventory as well as underlying driving shock processes in terms of their deviations from the steady state. We write this solved model in a state space form with the effective inventory and the exogenous shocks as the states. Spot price is determined by the current states. Knowing the stochastic processes for the states, rational expectation of future spot price could also be attained. Appendix A offers more details on the solution algorithm.

To study the implication of the model, we calibrate the structural parameters in the model from the data or assign them according to literature estimates when available, and present the impulse response functions of price and effective inventory to different shocks under our parameterization. We show that an increase in effective inventory could accompany both an increase in spot price or expectation of future prices and a decrease in them. Persistent shock, temporary shock, and news shock each generates different profiles of dynamic responses in price, futures spread and effective inventory.

3.1 Parameterization

We consider the world market of oil as competitive and its representative firm as modeled earlier. The net marginal inventory cost function MIC of the representative firm is crucial to price-inventory dynamics. However, the literature is limited on its key parameters ϕ in the marginal convenience yield function, the marginal physical storage cost δ, and the marginal inventory adjustment cost Δ'. We follow Pindyck (1994)'s estimate for heating oil and set $\phi = 1.42$, $\delta = 0.89$. We arbitrarily set $\Delta' = 0.2$.

The world demand for oil depends on a key parameter γ, the short-run price elasticity of demand for crude oil, whose estimate in the literature ranges from 0.05 to 0.44 (Dahl (1993), Cooper (2003), Baumeister and Peersman (2012), Bodenstein and Guerrieri (2011), Kilian and Murphy (2010))[2]. We pick an average value for γ, 0.25.

The relative supply of oil is described by the exogenous processes defined in Equations 5, 7, 8 and 9. For the shocks, we set the autoregressive coefficient for the persistent shock ρ^τ to

[2] See Hamilton (2009a) for a summary of the estimates in the literature in Table 1. Kilian and Murphy (2010) also provides a brief survey of the estimates.

be 0.9, the temporary shock ρ^c 0.1, and the news shock ρ^{n_τ} 0.9. Their standard deviations are set to 1.

The baseline parameterization is summarized in Table 1.

3.2 Simulated Impulse Response Functions

To obtain a better picture of what the model implies about the short-run price-inventory dynamics, we examine how oil markets respond to a variety of shocks to the relative supply, including persistent shock, temporary shock and news shock by simulating their impulse response functions under the model parameterization.

As the plottings in Figure 1 show, overall, the impulse response functions to persistent and temporary shocks are similar. A positive persistent shock to relative supply causes a price drop that is larger in scale and longer in duration compared to a temporary one. The changes in futures spread are negative and effective inventory positive following both shocks.

More specifically, as for futures spread in the second column, $P_t - E(P_{t+1})$, the effect of a persistent shock is more lasting than a temporary one, but smaller in scale, indicating that a persistent shock causes a similar drop in expectation of future price to the spot price decrease, while a temporary shock doesn't affect the former much. The initial dip in the futures spread in response to a persistent shock is due to the different recovering speed of spot price and expectation of future price after the shock. Overall, both shocks cause the spot price to drop more than the expectation of future price. The news shock instead causes the expectation of future price to drop more than the spot price for the early 10 periods, and the futures spread changes its sign after later.

The effects on effective inventory decision n_{t+1} is similar to futures spread. A positive persistent shock and a temporary shock both cause an immediate accumulation in effective inventory, which is drawn out later depending on the persistence of the shocks. The accumulation after a persistent shock is smaller compare to after a temporary shock. However, after a news shock of the same size, the effective inventory is drawn down by a larger amount, with both spot and expectation of future prices experiencing decreases. The intuition behind this observation is that, expecting a future glut of supply, market participants start to draw out from the inventory right away. The release of inventory effectively increases the oil available for consumption and lower the spot price. On the surface we observe decreases in all variables: spot price, expectation of future price and inventory. In the other two scenarios, inventory is accumulated passively as a result of increased supply and so is the price drop. On the surface we observe decreases in prices accompanied by inventory accumulation. The different profiles of price-inventory dynamics will enable us to sort out the different shock processes driving behind them.

To summarize, the profiles of the dynamic response of prices and inventory are very different in response to the shocks. The news shock of the same size as other two shocks generates much larger impulse responses, especially in spot price. In the next section, we will bring the model to data and study the forces behind the recent oil price fluctuations.

4 Data and Model Estimation

In this section we present the estimation of the model using monthly data on prices, inventory and world supply from 1988 Jan to 2011 April. For prices, we use real spot and futures (1-month and 6-month) prices of WTI deflated by monthly CPI (1982-84=100). While world inventory of crude oil is not available, we use OECD inventory instead as its proxy, which is end-of-month US commercial inventory of crude oil scaled by the ratio of OECD to US petroleum products stock, following Hamilton (2009a) and Kilian and Murphy (2010). The world supply of crude oil is available from Energy Information Administration (EIA).

As mentioned earlier, the trend in world supply and possibly inventory need to be treated. The effective inventory n_{t+1} is attained by taking the ratio of OECD inventory and world supply. Supply growth rate $\log(\mu_t^s)$ is also available by taking the log difference in world supply. We adjust the seasonality in the effective inventory data regressing the data on monthly dummies.

The data for estimation include: real spot and futures (1-month and 6-month) prices of WTI, seasonally-adjusted OECD effective inventory, and world supply growth rate, which are presented in Figure 2.

4.1 Observable State Variables

One advantage of estimating the model is that some of the state variables are observed. Both effective inventory n_t and world supply growth rate μ_t^s are available from the available data .However, due to the lack of data on inventory at world level, the OECD effective inventory is just a proxy with possible error. As a result, the observation equation for effective inventory differs from its state-equation form in the extra measurement error ϵ_t^n. On the other hand, the world supply growth rate shock $\{\mu_t^s\}$ is fully observed without any measurement error. Its observation equation is then just an identity mapping the data to the state variable in the model. This gives us two additional observation equations in the state-space form:

$$\begin{bmatrix} \hat{n}_t \\ \hat{\mu}_t^s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ \hat{\mu}_t^s \\ y_t^\tau \\ y_t^c \\ n_t^\tau \end{bmatrix} + \begin{bmatrix} \epsilon_t^n \\ 0 \end{bmatrix} \quad \epsilon_t^n \sim N(0, \sigma_n^2) \tag{13}$$

4.2 Estimation Results

Table 2 and 3 summarize the estimation results. Aside from δ, all estimates are significant at 99% confidence level. Estimates of the monthly dummies indicate that effective inventory tend to be lower during colder months than warmer months. We identify a highly persistent shock, a white-noise temporary shock and a random-walk news shock behind the price-inventory dynamics. The autoregressive coefficient of the persistent shock is as high as 0.94. With the estimates, we first study the impulse response functions, as presented in Figure 3.

Figure 3 plots the impulse response functions of the price and inventory to the shocks. In response to a one-standard deviation persistent shock, spot price drops by 10 percent. Expected future spot price drops by even a little more. Effective inventory accumulates right away, and peaks at 7 percent after 2 years. A temporary shock of the same size causes a smaller drop of 2 percent in price which dissipates quickly. Expected future spot price is barely affected. The accumulation in effective inventory is close to zero compared to the persistent case and the initial accumulation is drawn down right away. Overall the change in effective inventory is close to nothing. The news shock instead causes no contemporaneous change in spot price, a slightly smaller drop in the expected future spot price, and a depletion of effective inventory that looks like the opposite to the persistent case.

Overall, the impulse response functions from our estimates show that in general the shock effects on effective inventory is small but persisting. Persistent shock and news shock generate very similar responses in effective inventory, but different responses in price to accompany. The persistence of shocks shows more through the price responses. It's with these different price-inventory dynamics that we are able to sort out the different shocks over the sample period.

Figure 4 then plots the smoothed states from the estimates with 90% CI's. As explained earlier, effective inventory is treated as an endogenous state variable observed with measurement error. The smoothed effective inventory is close to the original data nonetheless, and the estimate of the measurement error standard deviation is small (0.001). An interesting observation is that the news shock seem to experience two stages through out the sample period. Prior to 2004, the news shock is positive in general. At around 2004, the news shock experience a sudden drop and it stays negative for most of the time. Also, the tight confidence intervals show that the smoothed states are not sensitive to the uncertainty of the parameter estimates.

4.3 Historical Decompositions

Figure 5 presents the contribution of different shocks to effective inventory and oil prices respectively, to show the relative importance of the shocks to the observables.

(a) shows the effective inventory had there been only one shock and the same initial effective inventory stock with 90% CI. In the observation equation for effective inventory, the autoregressive coefficient on itself is 0.9946. As a result, inventory would naturally decrease over time had there been no shock. The contribution of temporary shock is minimal, and doesn't divert inventory from its natural course. The contribution of supply growth rate shock is similar, though it adds more wiggles to the natural course. In the presence of only persistent shock, effective inventory steadily accumulates until around 1999, when the rapid depletion starts. Earlier impulse response function analysis shows that persistent shock has almost permanent effect on effective inventory. Its accumulated effect on inventory reflects abundant supply relative to demand prior to 2000, and much more constrained relative supply in the 2000s. With only the news shock, effective inventory shows almost opposite time path compared to persistent shock, caused by some dramatic change in the long-term expectation from abundant to limited relative supply. Judging by their scale, the effective inventory accumulation caused by change in expectation is less than the depletion due to the persistent shock. In face with strong global demand and staggered world production, the spontaneous need to draw down the inventory due to limited relative supply dominates the accumulation caused by speculative incentive, which leads to lower relative inventory than its natural course. Similarly, Hamilton (2009a) discusses the trend in the inventory by comparing the current inventory to a historical 17-year average, and concludes that the inventory is significantly lower than normal. Given the low price elasticity, the shock effects on inventory would be small in general. To understand their contribution to price, we need to decompose the shock contribution to the prices.

(b), (c) and (d) show the historical decomposition of shock effects on prices respectively. The contribution of the persistent shock seems to be the major part in all three prices. Contribution from temporary shock and supply growth rate shock are minimal. The news shock causes the spot price to increase from steady state by around 1 percent starting from 2004. Its resulting increases in futures prices are even higher. However, the overall contribution of news shock to spot price is about 1% of persistent shock, less than 10% for 1-month futures price and 40% for 6-month futures prices. And the news shock contribution to spot price has relatively wide confidence interval.

To summarize, both persistent shock and news shock contribute importantly to effective inventory. The persistent shock appears to be the major driving force for prices, though news shock matters more for futures price with longer maturity term.

4.4 Robustness of the Results

One concern is that the specific functional forms for the marginal convenience yield and marginal inventory adjustment cost may over restrict the model. Since the model is log-linearized, the state-space system brought to estimation is essentially a linear mapping from the last-period effective inventory and three current shocks to current inventory and prices. The specific functions and its deeper parameters impose constraints about the relation among the loading factors in the mapping matrices. We check the robustness of the results by directly estimating the loading factors without constraints.

Under the more relaxed setting, the persistent shock is estimated to have an autoregressive coefficient of 0.94 and standard deviation 0.02, temporary shock 0.02 and 0.005, news shock 1 and 0.005, which are not significantly different from earlier estimates. The smoothed states indicate that the results are not driven by the specific functional forms used.

5 Forecasting

One potential of the model is to forecasting oil spot price using past data on both price and effective inventory. We evaluate the forecasting ability of the model by carrying out out-of-sample forecast for 30 periods and compare the forecasts to a random walk and futures price of the same forecasting horizon. In specific, we look at the cases of 1-month, 2-month, 3-month, 6-month, 8-month and 12-month ahead forecasting of spot price. The reason for different forecasting horizon is that the model is estimated using 1-month and 6-month futures prices. To really compare the forecasting ability of the model with futures prices, we need to consider forecasting horizon other than the maturity term of the futures prices.

Table 4 presents the summarized the forecasts statistics. Figure 6 shows the out-of-sample forecasts against the competitors and data. In both 1-month and 6-month out-of-sample forecasts, the model outperforms a random walk, and performs as well as corresponding futures prices. In 2-month and 3-month cases, the model outperforms a random walk, and is outperformed by corresponding futures prices. In 8-month and 12-month cases, however, the model outperforms both random walk and corresponding futures prices.

6 Conclusion

In this paper, we attempt to better understand the price dynamics in world crude oil market with the inventory data. To do this, we consider the competitive inventory-holding decision of oil producers. Price and inventory decision are endogenously determined and made, under current and expected future market condition. We're especially interested in the ability of such a framework to uncover the underlying the market condition and expectation from the observed price and inventory data. Parameterization of the model shows that theoretically the price-inventory responses to different shocks are different. Namely, in response to traditional contemporaneous shocks to relative supply (consider positive shock), spot price drops and inventory accumulates. In response to news shock, spot price drops and inventory is drawn down. It's the different response profiles that enable us to sort out the different shocks from the data. Our estimates using crude oil data show that, the market is characterized by persisting constrained world supply relative to demand in recent years, accompanied by the long-term expectation of continuing constraints on our supply. The role of expectation in driving up the spot price is limited though.

A Solving the Model

The solution of the detrended model involves firstly finding its steady state and log-linearizing the model around the steady state, and secondly solving the linear system using Blanchard and Kahn (1980) and writing the model in a state-space form. We use data to help calibrate some parameters and estimates from the literature for others in our baseline simulation.

First, we write out the model in steady state:

$$1 = \beta - [\alpha(\frac{n/\mu^s}{n/\mu^s + 1 - n})^{-\phi} + \delta] \tag{14}$$

$$P = c[(n/\mu^s + 1 - n) * q^s]^{-\frac{1}{\gamma}} \tag{15}$$

$$\log \mu^s = \bar{\mu} \tag{16}$$

$$\log q^s = \bar{y} \tag{17}$$

$$y^\tau = 0 \tag{18}$$

$$y^c = 0 \tag{19}$$

$$n^\tau = 0 \tag{20}$$

$$n^c = 0 \tag{21}$$

Then we log-linearize the model around the steady state:

$$\hat{P}_t = \beta E_t[\hat{P}_{t+1}] - \frac{MIC}{P} E_t[M\hat{I}C_{t+1}] \tag{22}$$

$$M\hat{I}C_{t+1} = \hat{P}_t + micn_0 \hat{n}_t + micn_1 \hat{n}_{t+1} + micn_2 \hat{n}_{t+2} + micu_0 \hat{\mu}^s_t + micu_1 \hat{\mu}^s_{t+1} \tag{23}$$

where

$$micn_0 = -\frac{1}{\beta - 1} * \Delta' * \mu^s \tag{24}$$
$$micn_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{1 - n}{n/\mu^s + 1 - n} + (1 + \beta) * \Delta' * \mu^s] \tag{25}$$
$$micn_2 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{n}{n/\mu^s + 1 - n} - \beta * \Delta' * \mu^s] \tag{26}$$
$$micu_0 = \frac{1}{\beta - 1} * \Delta' * \mu^s \tag{27}$$
$$micu_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{n - 1}{n/\mu^s + 1 - n} - \beta * \Delta' * \mu^s] \tag{28}$$

$$\hat{P}_t = -\frac{1}{\gamma}[pn_0 \hat{n}_t - pn_1 \hat{n}_{t+1} - pu \hat{\mu}^s_t + py \hat{q}^s_t] \tag{29}$$

where

$$pn_0 = \frac{n/\mu^s}{n/\mu^s + 1 - n} \tag{30}$$
$$pn_1 = \frac{n}{n/\mu^s + 1 - n} \tag{31}$$
$$pu = \frac{n/\mu^s}{n/\mu^s + 1 - n} \tag{32}$$
$$py = 1 \tag{33}$$

We could write the log-linearized model in terms of state variables X_t, costate variables Y_t and exogenous shock variables e_t:

$$X_t = \begin{bmatrix} \hat{n}_t \\ \hat{n}_{t+1} \end{bmatrix}, \; Y_t = \begin{bmatrix} \hat{P}_t \end{bmatrix}, \; e_t = \begin{bmatrix} \hat{\mu}_t^s \\ y_t^\tau \\ y_t^c \\ n_t^\tau \end{bmatrix}$$

Using Blanchard and Kahn (1980), we solve for the following state-space form of the model, where the state variables are n_t, μ_t^s, y_t^τ, y_t^c, n_t^τ, and the observed variable is P_t.

State equation:

$$\begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} = \begin{bmatrix} F_{n,n} & F_{n,e} \\ F_{e,n} & F_{e,e} \end{bmatrix} \begin{bmatrix} \hat{n}_{t-1} \\ e_{t-1} \end{bmatrix} + Z * v_t \qquad v_t \sim N(0, U) \tag{34}$$

where $v_t' = \begin{bmatrix} \epsilon_t^\mu & \epsilon_t^{y_\tau} & \epsilon_t^{y_c} & \epsilon_t^{n_\tau} \end{bmatrix}$, $Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} \sigma_\mu^2 & 0 & 0 & 0 \\ 0 & \sigma_{y_\tau}^2 & 0 & 0 \\ 0 & 0 & \sigma_{y_c}^2 & 0 \\ 0 & 0 & 0 & \sigma_{n_\tau}^2 \end{bmatrix}$.

Observation equation:

$$\hat{P}_t = \begin{bmatrix} H_{P,n} & H_{P,e} \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} \tag{35}$$

Table 1: **Model Parameterization**

Parameters	Value	Description
β	0.997	monthly depreciation rate
γ	0.25	price elasticity of demand for crude oil
ϕ	1.42	parameter in net marginal convenience yield
Δ'	0.2	marginal cost of inventory change
δ	0.89	marginal physical storage cost
ρ^{τ}	0.9	autoregressive coefficient of persistent shock
ρ^{c}	0.1	autoregressive coefficient of temporary shock
$\rho^{n_{\tau}}$	0.9	autoregressive coefficient of news shock
$\sigma_{y_{\tau}}$	1	s.d. of persistent shock
$\sigma_{y_{c}}$	1	s.d. of temporary shock
$\sigma_{n_{\tau}}$	1	s.d. of news shock
$\sigma_{\mu^{s}}$	1	s.d. of shock

Table 2: **Estimated Model for Crude Oil Market**

Parameters	Value	Std. Error	Description
β (set)	0.997		monthly depreciation rate
γ (set)	0.25		price elasticity of demand for crude oil
ϕ (set)	1.42		parameter in net marginal convenience yield
Δ'	125.27^{***}	(35.31)	marginal cost of inventory change
δ	0.001	(0.002)	marginal physical storage cost
ρ^{τ}	0.94^{***}	(0.264)	autoregressive coefficient of persistent shock
ρ^{c}	0.00^{***}	(0.000)	autoregressive coefficient of temporary shock
ρ^{n_τ}	1.00^{***}	(0.287)	autoregressive coefficient of news shock
σ_{y_τ}	0.022^{***}	(0.006)	s.d. of persistent shock
σ_{y_c}	0.005^{***}	(0.001)	s.d. of temporary shock
σ_{n_τ}	0.001^{***}	(0.000)	s.d. of news shock
σ_{μ^s} (set)	0.011		s.d. of news shock
$\sigma_{\hat{n}}$	0.001^{***}	(0.000)	s.d. of measurement error to effective inventory

Note: (i) Simulated standard errors of the estimates are in parentheses; (ii) *, ** and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

Table 3: **Estimated Model for Crude Oil Market - continued**

Parameters	Value	Std. Error	Description
Jan.	-0.04^{***}	(0.01)	monthly seasonality dummy
Feb.(set)	0		monthly seasonality dummy
Mar.	0.00	(0.01)	monthly seasonality dummy
Apr.	0.03^{***}	(0.01)	monthly seasonality dummy
May.	0.04^{***}	(0.01)	monthly seasonality dummy
Jun.	0.03^{***}	(0.01)	monthly seasonality dummy
Jul.	0.02^{***}	(0.01)	monthly seasonality dummy
Aug.	-0.00	(0.01)	monthly seasonality dummy
Sep.	-0.00^{***}	(0.01)	monthly seasonality dummy
Oct.	-0.03^{***}	(0.01)	monthly seasonality dummy
Nov.	-0.00	(0.01)	monthly seasonality dummy
Dec.	-0.01	(0.01)	monthly seasonality dummy

Note: (i) Simulated standard errors of the estimates are in parentheses; (ii) *, ** and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

Table 4: **Out-of-sample Forecast Performance**

(a) One-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	48.95	5.53	-0.0057
random walk	50.94	5.66	-0.0015
1-month futures price	48.95	5.53	-0.0057

(b) 6-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	0.0525	0.1626	0.0268
random walk	0.1626	0.2156	0.1035
6-month futures price	0.0525	0.1626	0.0268

(c) 2-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	0.0368	0.1182	-0.0173
random walk	0.0398	0.1272	0.0022
2-month futures price	0.0356	0.1146	-0.0223

(d) 3-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	0.0602	0.1459	-0.0171
random walk	0.0684	0.1695	0.0170
3-month	0.0601	0.1492	-0.0078

(e) 8-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	0.0350	0.1323	0.0464
random walk	0.0845	0.2140	0.1489
8-month futures price	0.0384	0.1391	0.0540

(f) 12-month ahead Out-of-sample Forecast

forecaster	MSE	MAE	ME
model	0.0257	0.1191	0.0614
random walk	0.1087	0.2572	0.2118
12-month futures price	0.0353	0.1385	0.0814

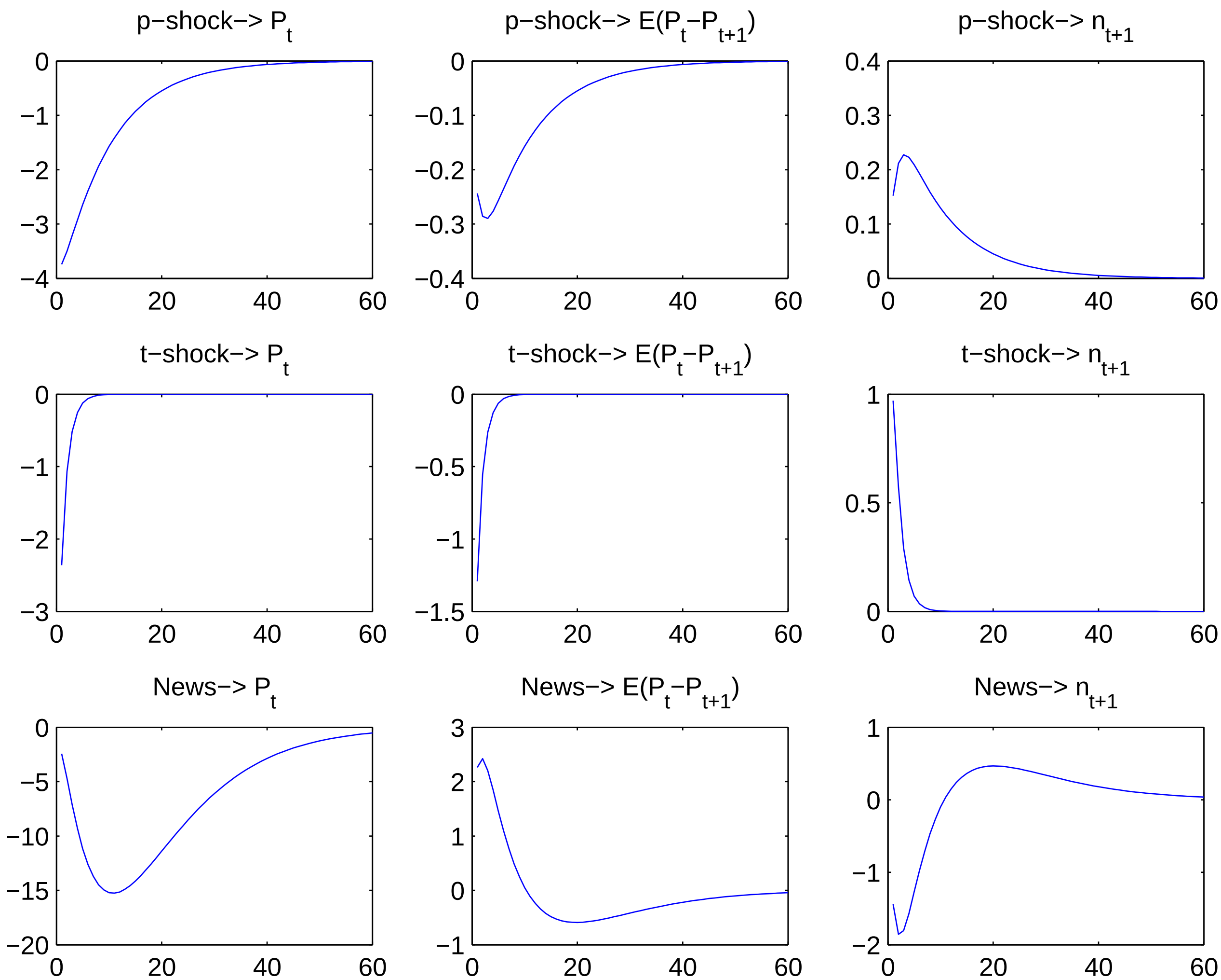
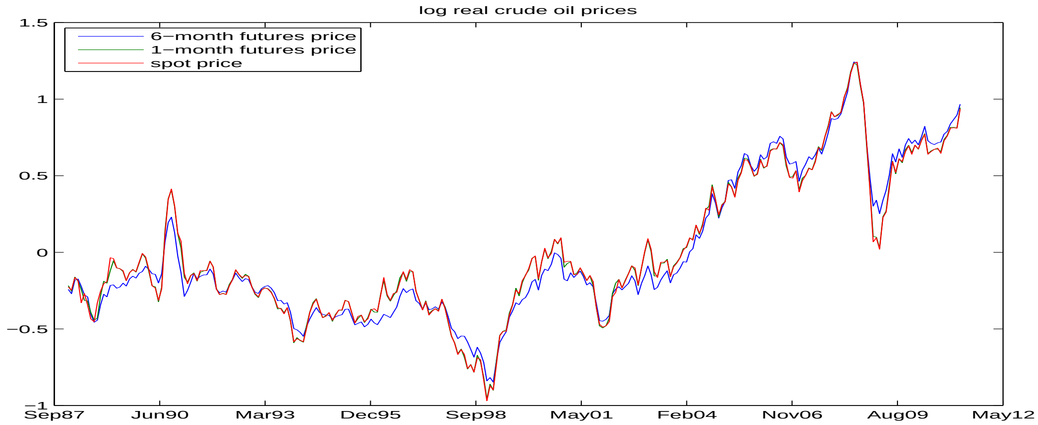
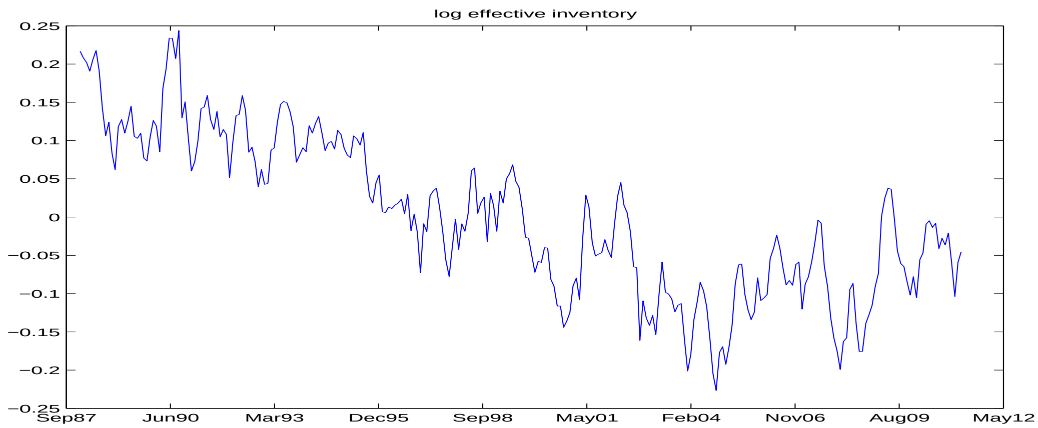


Figure 1: Impulse Response Functions

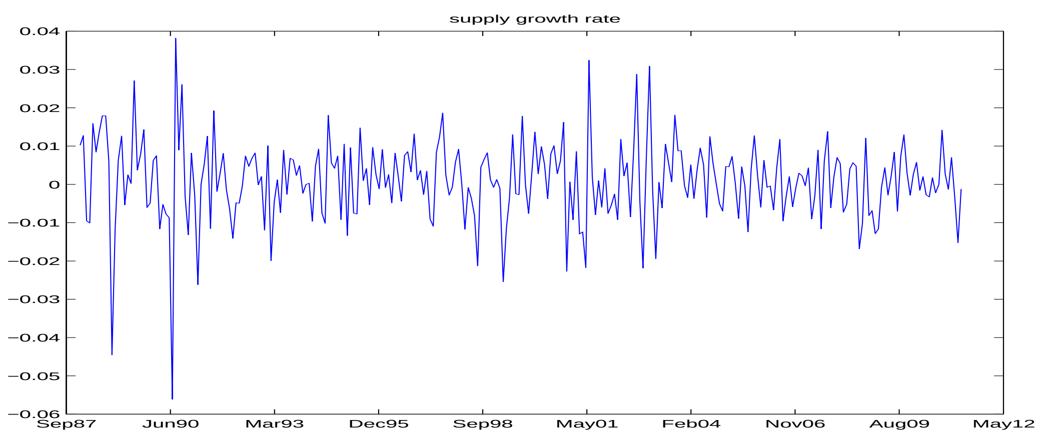
Note: p-shock: persistent shock; t-shock: temporary shock; news: news shock



(a) WTI Prices



(b) Effective Inventory



(c) Supply Growth Rate

Figure 2: Data Overview

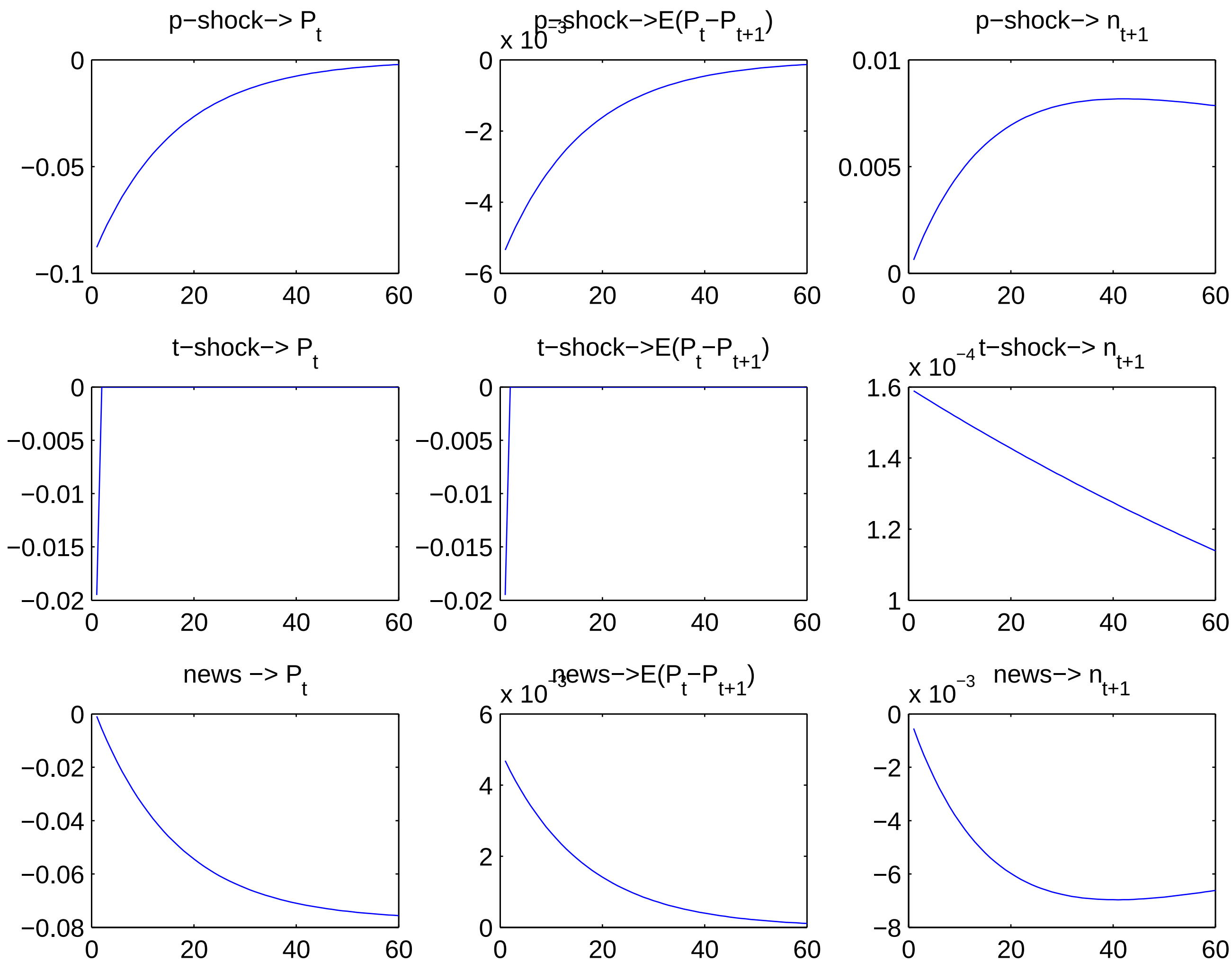


Figure 3: Estimated Impulse Response Functions

Note: p-shock: persistent shock; t-shock: temporary shock; news: news shock

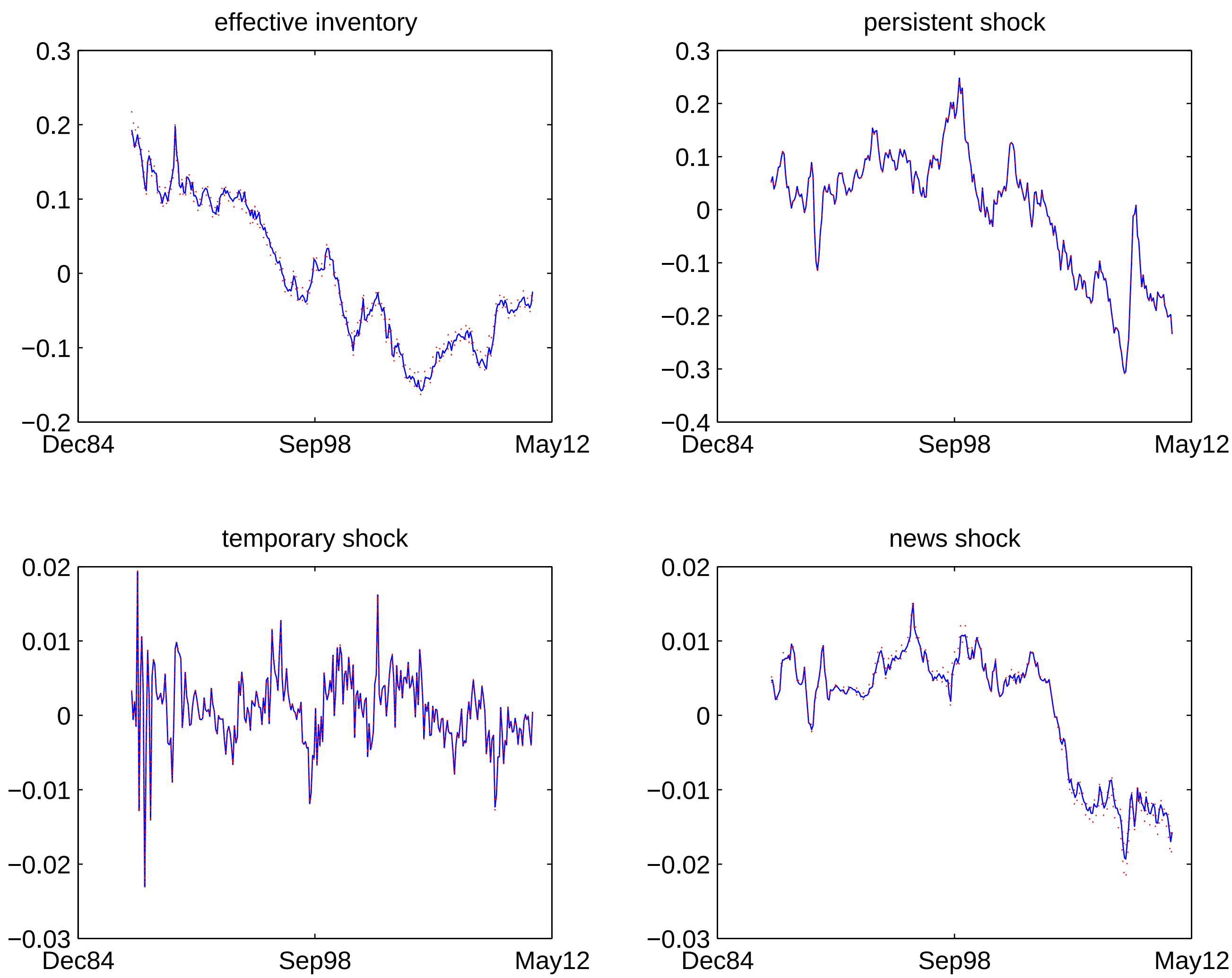
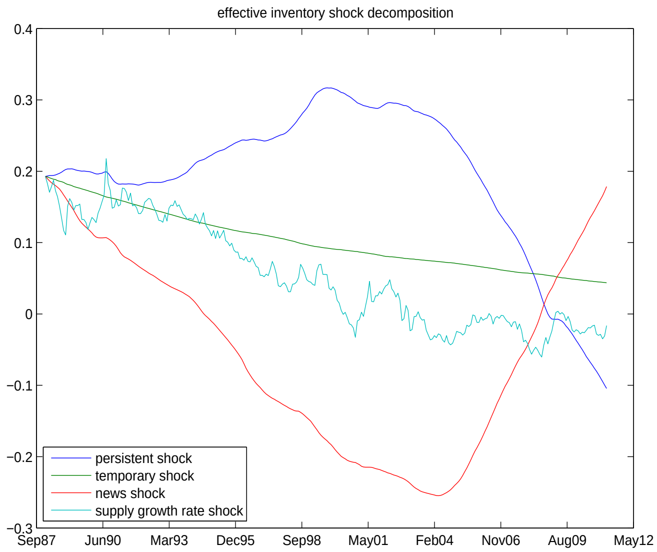
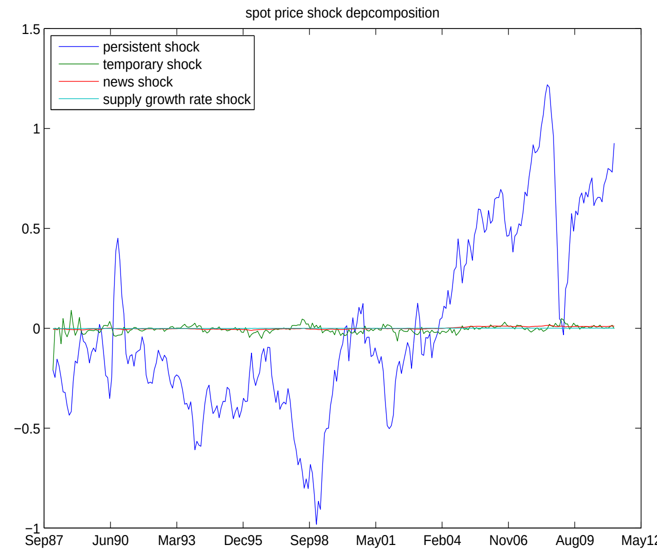


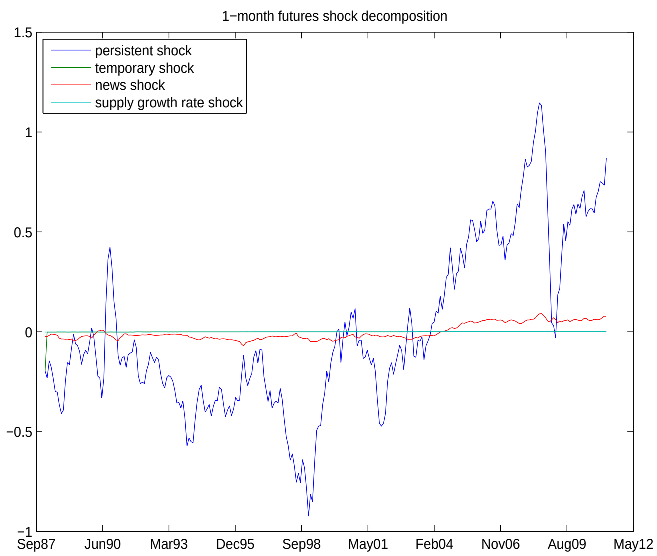
Figure 4: Kalman Smoothed States with 90% CI



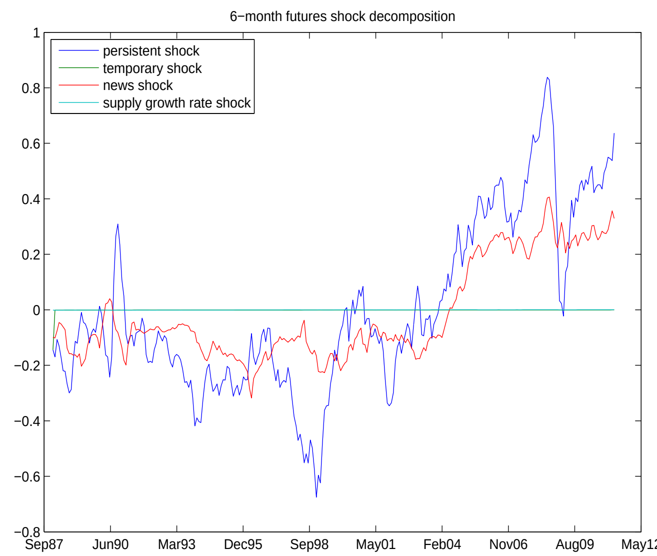
(a) Effective Inventory



(b) Spot Price

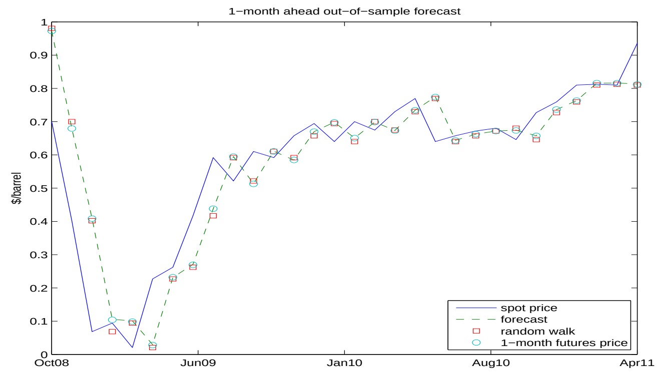


(c) 1-month Futures Price

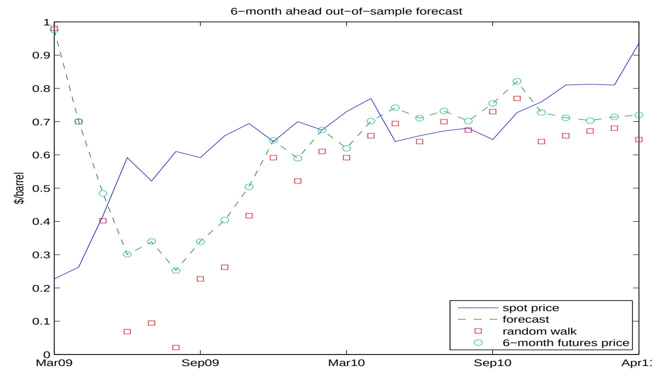


(d) 6-month Futures Price

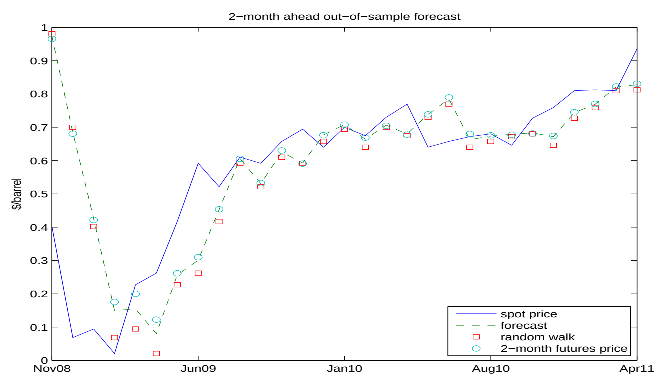
Figure 5: Shock Decomposition Overview



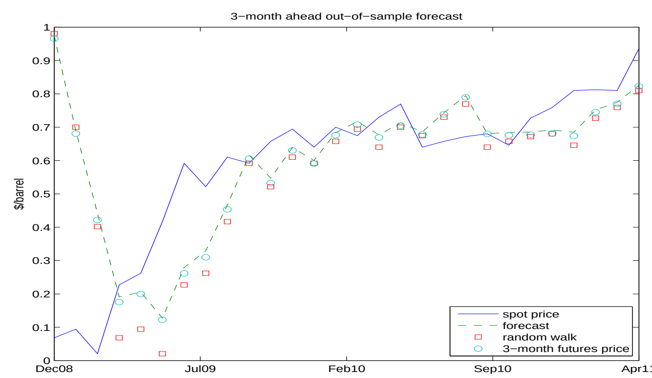
(a) 1-month ahead



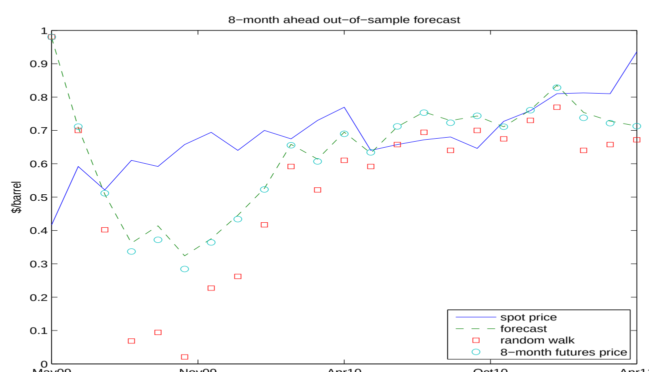
(b) 6-month ahead



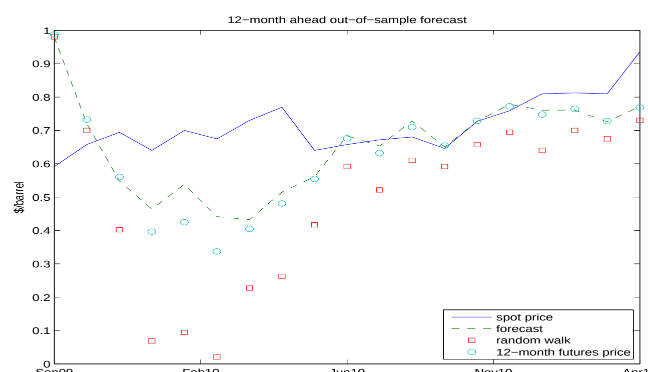
(c) 2-month ahead



(d) 3-month ahead



(e) 8-month ahead



(f) 12-month ahead

Figure 6: Out-of-Sample Forecast

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